An Introduction to Time-weighted vs. Money-weighted Returns

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1 Motivation

The Canadian Securities Administrators (CSA) has recently mandated that performance reporting on client statements use money-weighted returns, to be implemented by 2016. Most firms, including Steadyhand, currently report client account performance using time-weighted rate of returns.

In this article I will summarize the two methods and provide an example of when the resulting performance numbers can be quite different. I've tried to minimize the use of math, but it really is essential in understanding how the methodologies work.

2 What is Rate of Return?

Return is essentially the change in value of your portfolio over time. If we ignore cash flows in or out of the account, it can simply be viewed as the gain or loss in market value relative to the beginning market value.

If we let R represent the return, and MV represent the account or portfolio's market value:

$$R = (MV_{end} - MV_{begin})/MV_{begin}$$

The total return over a period can be determined by compounding the series of returns in each subperiod:

$$R_{period} = ((1 + R_1) * (1 + R_2)...(1 + R_n)) - 1$$

And finally, we can annualize the return over *n* years using the formula:

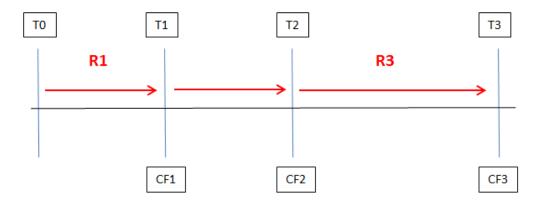
$$R_{annualized} = \sqrt[n]{(1 + R_{multiyear})} - 1$$

In reality, almost all accounts have cash flows as clients purchase or redeem units. There are two broad classes of calculating performance in the face of these cash flows; *time-weighted*, and *money-weighted*.

3 Time-weighted Rate of Return

The Time-weighted Rate of Return methodology (TWRR) determines the performance of the account over the time invested, while factoring out cash flows and the relative size of the account during each performance sub-period.

To begin, the performance period is divided into sub-periods ending on the day of each cash flow. In the diagram below, the period has been divided into three sub-periods corresponding to the cash flows:



The return for each sub-period is calculated by taking the ending market value of the sub-period and subtracting the net cashflow *CF*, leaving the pre-cashflow ending market value to compare against the beginning market value:

$$R_{subperiod} = (MV_{end} - CF - MV_{begin})/MV_{begin}$$

As described earlier, the returns for the total period are determined by geometrically linking the return for each sub-period:

$$R_{period} = ((1 + R_1) * (1 + R_2) * (1 + R_3)) - 1$$

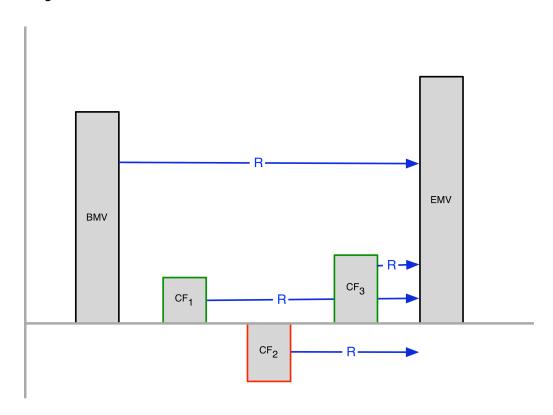
From the asset manager's perspective, TWRR is often preferred because it shows the performance of the underlying funds (i.e. the manager's performance), while factoring out the decisions made by the client with respect to timing of contributions and withdrawals (which are beyond the asset manager's control).

However, TWRR may not accurately reflect the return experienced by the client, either due to their decisions, or their investment advisor's decisions.

4 Money-weighted Rate of Return

Money-weighted Rate of Return, also known as the Internal Rate of Return (IRR), determines the rate of return of an investment while factoring in the cashflows and relative size of the account over the time period.

The IRR is the discount rate that results in the ending market value being equal to the sum of the beginning market value and each of the net contributions, all compounded with that rate over their respective durations. I find it easiest to understand when looking at the following representation, which shows the beginning market value and each cashflow all compounded at the rate R to result in the ending market value:



$$EMV = BMV(1+R) + CF_1(1+R)^{W_1} + CF_2(1+R)^{W_2} + CF_3(1+R)^{W_3}$$

where

$$W_i = (CD - D_i)/CD$$

 $CD =$ calendar days in period

 $D_i =$ days since beginning of period

The IRR is a more accurate representation of the returns that the client actually experiences.

4.1 Calculating IRR

Unlike TWRR, IRR cannot be solved directly via formula. Instead, it is solved iteratively by guessing an initial value of R, calculating the resulting ending market value, and then adjusting R and repeating until the calculated ending market value gets very close to the actual. For the math geeks, the Newton-Raphson method is commonly used to solve for R:

$$\sum_{i=1}^{n} CF_{i}(1+R)^{W_{i}} - MV_{ending} = 0$$

For learning purposes, we can also use Microsoft Excel's XIRR function to solve the equation. The Excel help file describes the XIRR function as "Returns the internal rate of return for a schedule of cash flows that is not necessarily periodic."

Note that the IRR is an annualized rate of return.

5 An Example of Widely Differing Results

In cases where there are no subsequent cashflows after the initial investment, or when the cashflows are small relative to the initial investment, the IRR and TWRR methodologies will return similar results. However, when there are large cashflows relative to the initial investment amount, the results can be guite different.

The following scenario is from a real client account opened in March 2009, with the performance reporting period ending September 2010.

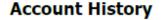
The Account Activity table on the client statement shows the since inception activity:

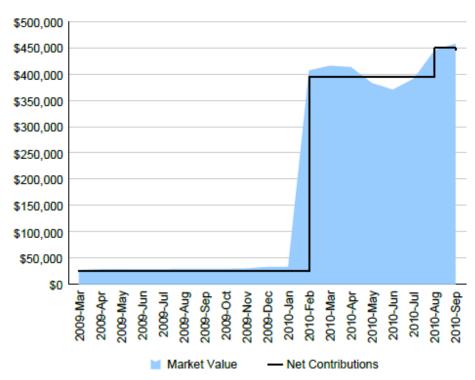
	amount
beginning market value	25,000.00
contributions	425,000.00
redemptions	(5,000.00)
gain/loss	12,970.02
ending market value	457,970.02

The TWRR for the account over the period is **20%**. From the client's perspective this is not very intuitive; they have invested a net amount of \$445,000 and realized a gain of approximately \$13,000 over eighteen months, which doesn't appear anywhere near a return of 20% (clients typically do a mental calculation of dividing 13/445, resulting in a total return of about 3%).

The reason, of course, is that the TWRR methodology factors out the cashflows and much of that return was realized in 2009 when the amount invested was only \$25,000. Incidentally, March 2009 was the market low after the 2008 financial crisis, so this was an excellent time to invest.

The timing of the cashflows is best shown on the following Account History chart:





The calculated IRR for the account since inception is **4.9%**, significantly less than the TWRR.

We can visualize the calculation of the IRR calculation with the following table, which shows each cashflow, the number of days it was in the performance period (cf-days), and the resulting contribution to the ending value (irr-cashflow). Note that the ending market value is shown as a negative cashflow.

date	amount	cf-day s	irr-cashflow
20090309	25,000.00	570	26,928.62
20100222	370,000.00	220	380,766.21
20100820	50,000.00	41	50,267.99
20100827	5,000.00	34	5,022.21
20100907	-5,000.00	23	-5,015.02
20100930	-457,970.02	0	-457,970.02
total irr-cashflow			0.0

From an intuitive perspective, the 4.9% return matches more closely with the returns experienced by the client.

6 Money-weighted Returns at Steadyhand

As we have seen above, in the presence of large cashflows the TWRR and IRR performance numbers can diverge quite dramatically.

While we have until 2016 to implement money-weighted returns, we are beginning the process of transitioning now. We currently have an internal (i.e. unofficial) system calculating IRR for all client accounts so that we can better understand the implications. In addition, we are working with our vendors to ensure that they are ready for the transition.

Most of our clients do not have a grasp of TWRR (understandably), let alone IRR, so we are beginning to think about the education process, including articles like this.

We don't anticipate that clients will see money-weighted returns on their statements until 2015.